JHEP HYPER VERSION

Received: *July 09, 2001*, Accepted: *July 20, 2001*

# Excitations on wedge states and on the sliver

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JHEP07(2001)024

Abstract: We study ghost number one excitations on the sliver to investigate the solution of string field actions around the tachyon vacuum. The generalized gluing and resmoothing theorem is used to develop a method for evaluating the effective action for excitations on both the wedge states and the sliver state. We analyze the discrete symmetries of the resulting effective action for excitations on the sliver. The gauge unfixed effective action till level two excitations on the sliver is evaluated. This is done for the case with the BRST operator $c_0$ and $c_0 + (c_2 + c_{-2})/2$ with excitations purely in the ghost sector. We find that the values of the effective potential at the local maximum lie close by for the zeroth and the second level of approximation. This indicates that level truncation in string field theory around the tachyon vacuum using excitations on the sliver converges for both choices of the BRST operator. It also provides evidence for the conjectured string field theory actions around the tachyon vacuum.

Keywords: Bosonic Strings, D-branes, Conformal Field Models in String Theory.

## Contents





## 1. Introduction

The phenomenon of tachyon condensation has proved to be a valuable guide in exploring various open string field theories. In this paper we will restrict our focus to the cubic string field theory of Witten [1]. The tachyon in open bosonic string theory corresponds to the instability of the D25-brane to decay to the vacuum. It has been conjectured by Sen [2, 3] that there exists a stationary point in the tachyon potential.[1] The value of the tachyon potential at this stationary point should agree with the tension of the D25-brane. A co-dimension $p$-lump solution is conjectured to represent a D$(25-p)$-brane in the same theory. These conjectures have been verified to a remarkable degree of accuracy in cubic string field theory using level truncation as an approximation scheme [5]–[26]. The D25-brane decays to the vacuum when the tachyon condenses. Thus there should be no physical open string excitation around the tachyon vacuum. Recently strong numerical evidence for this conjecture was found in cubic string field theory in [27]. For a review of the study of tachyon condensation in bosonic and superstring field theories and other references see [28].

Lack of an analytical solution for the stable vacuum in cubic string field theory has made it difficult to study excitations around the vacuum. In [29] a class of string field theories were put forward as candidates for string field theory around the tachyon vacuum. These theories differ from the conventional cubic string theory of Witten in their choice of the BRST operator. In Witten's cubic string field theory, the BRST operator $Q_B$ is the usual one made of the ghost and matter stress energy tensor along with the world sheet $c$ and $b$ ghosts. In the candidates put forward in [29] the BRST operator $Q$ was chosen to be solely made of ghosts. Similar actions were derived from the purely cubic string field theory previously in [30]. The BRST operators considered in [29] has the advantage of having no manifest physical states. The BRST operators chosen had trivial cohomology. These operators also preserved the gauge invariance of the cubic action.

In [31] classical solutions of string field theory around the tachyon vacuum was studied. These solutions factorised as a tensor product of the ghost sector and the matter. Assuming factorization the matter sector of the string field equation reduces to

$$\Psi * \Psi = \Psi \,. \tag{1.1}$$

Here $\Psi$ is a string field and the star product is taken over the matter sector. Recently generalizations to the solution of the above equation was constructed using the projection operator method in [32, 33, 34].[2] In [36] solutions to (1.1) was constructed for a general boundary conformal field theory. The translational invariant solution to (1.1) was constructed using the explicit representations of the star product in terms of Neumann functions [37, 38, 39] in [18]. The matter sector solutions reproduced the ratio of D-brane tensions to a high degree of accuracy.

In this paper we explore the the solution of the string field equations around the tachyon vacuum including the ghost sector. We focus on the translational invariant solution which should correspond to the D25-brane. The translational invariant solution of (1.1) was shown to be the matter sector of the sliver state constructed in [13]

[1] For earlier studies of the tachyon in open string theory see [4].

[2] It is interesting to note that this equation and its solution using the projection operator method has appeared earlier in the context of $c = 1$ string field theory in [35].

This prompts us to study ghost number one excitations on the sliver state with the hope of solving the string field equation around the tachyon vacuum. Assuming the solution of the string field theory equation corresponding to the D25-brane factorizes to the matter and the ghost sector the excitations can be chosen to be purely in the ghost sector. To evaluate the effective action of these excitations we need to evaluate the Witten vertex and the quadratic term of these excitations. The sliver state is given by

$$\begin{aligned} |\Xi\rangle &= \exp\left(-\frac{1}{3}L_{-2} + \frac{1}{30}L_{-4} - \frac{11}{1890}L_{-6} + \frac{34}{467775}L_{-8} + \cdots\right)|0\rangle \\ &= U|0\rangle\,. \end{aligned} \tag{1.2}$$

Here $L_n$'s are the combined matter and ghost Virasoro generators. The sliver state consists of infinite number of levels. Conventional methods of evaluating the action is therefore difficult.

We use the generalized gluing and resmoothing theorem [40, 41] to develop a method for evaluating the effective action of excitations on the sliver state.[3] We mainly analyse excitations of the form

$$\tilde{\Phi}(0) = U\Phi(0)U^{-1} \tag{1.3}$$

on the sliver state. Here $U$ is defined in (1.2) and $\Phi(0)$ is the string field corresponding to the state $|\Phi\rangle$. The sliver state is a limit of the wedge state $|n\rangle$ as $n \to \infty$. Using the generalized gluing and resmoothing theorem we first develop the method to evaluate the Witten vertex and the kinetic term for excitations on the wedge state $|n\rangle$. Then we take the limit $n \to \infty$ to obtain the the Witten vertex and the kinetic term for excitations above the sliver state. We indicate when the limit is well defined.

We study the effective action of the excitations $\tilde{\Phi}(0)$ on the sliver state using both $Q = c_0$ and $Q = c_0 + (c_2 + c_{-2})/2$. These choices of BRST operators represent equivalent classes of BRST operators by field redefinitions of the kind $Q \sim e^K Q e^{-K}$. Here $K$ are conformal transformations which leave the string midpoint fixed. The operator $c_0$ denotes an equivalent class which does not annihilate the identity of the star algebra, while the operator $c_0 + (c_2 + c_{-2})/2$ does. We verify that the discrete symmetry properties of the effective action for excitations on the sliver state is inherited from the cubic action. We then evaluate the gauge unfixed action for fields till level 2 in $\Phi(0)$ and analyze its local maxima. The values of the effective potential at the local maximum lie close together for both the zeroth and second level approximation for the two choices of the BRST operator. This indicates that level truncation using excitations on the sliver state converges for both choices of the BRST operator. It is of interest to point out that level truncation in the string field theory around the tachyon vacuum with $Q = c_0$ seemed to push the maximum to zero [29].

[3] The author thanks Ashoke Sen for emphasizing the use of this theorem and pointing out ref. [41]

This paper is organized as follows. In section 2 we review the definition of the wedge states and the sliver state. In section 3 we review the generalized gluing and resmoothing theorem and use it obtain the product law of wedge state $|n\rangle * |m\rangle = |n+m-1\rangle$. In section 4 we use the generalized gluing and resmoothing theorem to obtain the kinetic term and the Witten vertex for excitations on the wedge states. Section 5 contains the quadratic term and the Witten vertex for excitations on the sliver state. The discrete symmetries of the effective action for excitations are analyzed. Then we analyze the effective action for both $Q = c_0$ and $Q = c_0 + (c_2 + c_{-2})/2$ till level 2 in $\Phi$. In section 6 we indicate the difficulty in generalizing the Witten vertex and the kinetic term for other kind of excitations. Section 7 contains our conclusions. Appendix A contains details of the expansion coefficients needed to write the sliver state in terms of ghost and matter oscillators. Appendix B contains the conformal transformation of non-primary fields involved in the effective action.

## 2. The wedge states and the sliver state

In this section we will review the definition of the wedge states introduced in [13]. The sliver state is then obtained as a limit of the wedge state. We construct the sliver sate in terms of matter and ghost oscillators as a squeezed state.

### 2.1 The definition of wedge states

In [13] the wedge states were defined as conformal transformation on the SL$(2,\mathbb{R})$ vacuum.

$$\langle n| = \langle 0|U_{W_n} \,, \tag{2.1}$$

where $\langle 0|$ stands for the left SL$(2,\mathbb{R})$ vacuum and $U_{W_n}$ denotes the conformal transformation corresponding to the map

$$W_n(z) = M\left[\left(\frac{1+iz}{1-iz}\right)^{2/n}\right]. \tag{2.2}$$

$M$ stands for any SL$(2,\mathbb{C})$ transformation which maps the unit circle to the real line. For instance we can take

$$M(z) = -i\frac{z-1}{z+1}\,. \tag{2.3}$$

The map $[(1+iz)/(1-iz)]^{2/n}$ takes the region inside the upper half disc $|z| \le 1$ to a wedge of angle $2\pi/n$. Hence the name wedge state. Given the above SL$(2,\mathbb{C})$ map $M(z)$ the function $W_n(z)$ is given by $\tan\left(\frac{2}{n}\tan^{-1}(z)\right)$. For the rest of the paper we will assume $M(z)$ is given by (2.3).

The operator $U_{W_n}$ corresponding to the map $W_n$ is written as

$$U_{W_n} = \exp(v_0 L_0)\exp\left(\sum_{n\ge 1} v_n L_n\right). \tag{2.4}$$



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Here $L_n$ are the combined matter and ghost Virasoro generators. The coefficients $v_n$ are obtained by comparing coefficients of different powers of $z$ on both sides of the equation

$$f(z) = \exp\left(\sum_{n\geq 1} v_n z^{n+1}\partial_z\right)\exp(v_0 z\partial_z)z\,. \tag{2.5}$$

Note that this function leaves the origin fixed. The ket $|n\rangle$ is given by

$$|n\rangle = U^{\dagger}_{W_n}|0\rangle\,, \tag{2.6}$$

where $U^{\dagger}_{W_n}$ is the BPZ conjugate of $U_{W_n}$. It is easy to show that $U^{\dagger}_{W_n} = U_{I\circ W_n^{-1}\circ I}$. Here $I$ denotes the $\mathrm{SL}(2,\mathbb{R})$ transformation $I(z) = -1/z$. The map $I\circ W_n^{-1}\circ I$ leaves the point at infinity fixed. Therefore we can write[4]

$$I\circ W_n^{-1}\circ I(z) = \exp\left(\sum_{n\leq -1} v'_n z^{n+1}\partial_z\right)\exp(v'_0 z\partial_z)z\,. \tag{2.7}$$

Once the coefficients $v'_n$ are determined from the above expansion the operator $U_{I\circ W_n^{-1}\circ I}$ can be constructed as

$$U_{I\circ W_n^{-1}\circ I} = \exp\left(\sum_{n\leq -1} v_n L_n\right)\exp(v_0 L_0)\,. \tag{2.8}$$

The wedge state $|1\rangle$ can be identified with the identity of the star algebra, while the state $|2\rangle$ is the $\mathrm{SL}(2,\mathbb{R})$ vacuum. It was shown in [13] that the wedge states obey the product law

$$|n\rangle * |m\rangle = |m+n-1\rangle\,. \tag{2.9}$$

We will re-derive this law by using the generalized gluing and resmoothing theorem explicitly. The sliver state is given as the limit

$$|\Xi\rangle = \lim_{n\to\infty}|n\rangle\,. \tag{2.10}$$

In terms of the explicit representation of the operator $U_{I\circ W_\infty^{-1}\circ I}$, there is a smooth limit given by

$$\begin{aligned}|\Xi\rangle = U_{I\circ W_\infty^{-1}\circ I}|0\rangle = \exp\left(-\frac{1}{3}L_{-2} + \frac{1}{30}L_{-4} - \frac{11}{1890}L_{-6} + \frac{34}{467775}L_{-8} + \cdots\right)\times \\ \times|0\rangle\,.\end{aligned} \tag{2.11}$$

Note that the dependence on $n$ drops out [13]. As the operator $U_{I\circ W_\infty^{-1}\circ I}$ operator commutes with the momentum the sliver state is a translational invariant state. Thus formally from (2.9) and (2.10) the sliver state satisfies

$$|\Xi\rangle * |\Xi\rangle = |\Xi\rangle\,. \tag{2.12}$$

[4]We use the symbol $\circ$ to denote composition of maps as well as the action of a conformal transformation on a field.



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## 2.2 Construction of the sliver state in terms of oscillators

It is useful to obtain the representation of the sliver state in terms of the matter and ghost oscillators. From (2.11) it is easy to see that the sliver state is created from the SL$(2,\mathbb{R})$ vacuum by exponentiation of an operator which is quadratic in the oscillators. Thus it is similar to a squeezed state. Let the translational invariant sliver state be expressed as a squeezed state by

$$\langle \Xi| = \langle 0| \exp\left(-\frac{1}{2}\eta_{\mu\nu}\alpha^{\mu}_{m} S_{mn}\alpha^{\nu}_{n} + c_s \tilde{S}_{si} b_i\right)\mathcal{N}\,, \tag{2.13}$$

where $m, n = 1, \ldots, \infty$, $i = 2, \ldots, \infty$ , $s = -1, \ldots, \infty$ and $\mu, \nu = 0, \ldots, 25$. $\mathcal{N}$ is a normalization constant which can be fixed from eq. (2.12). We will review the formulae to obtain the width matrix $S$ of the squeezed state, and then extend that method to obtain the ghost sector width $\tilde{S}$. From (2.13) we see that

$$\begin{aligned} S_{mn} &= -\frac{1}{mn}\langle \Xi|\alpha^{1}_{-m}\alpha^{1}_{-n}c_{-1}c_0c_1|0\rangle \\ &= -\frac{1}{mn}\langle 0|U_{W_\infty}\alpha^{1}_{-m}\alpha^{1}_{-n}c_{-1}c_0c_1U^{-1}_{\infty}|0\rangle\,. \end{aligned} \tag{2.14}$$

Here we have used $U^{-1}_{W_\infty}|0\rangle = |0\rangle$ the definition of the sliver state and the following commutation relation for the matter oscillators.

$$[\alpha^{\nu}_{n}, \alpha^{\mu}_{m}] = -n\eta^{\nu\mu}\delta(n+m)\,. \tag{2.15}$$

Similarly the width of the ghost sector is given by

$$\begin{aligned} \tilde{S}_{si} &= \langle \Xi|c_{-i}b_{-s}c_{-1}c_0c_1|0\rangle\,, \qquad \text{for } i \geq 2\,, \qquad s \geq 2 \\ &= \langle 0|U_{\infty}c_{-i}b_{-s}c_{-1}c_0c_1U^{-1}_{\infty}|0\rangle\,. \end{aligned} \tag{2.16}$$

Here we have used the anti-commutation relation $\{c_n, b_m\} = \delta(n+m)$. For $s = -1, 0, 1$ and $i \geq 2$ the width $\tilde{S}$ is given by

$$\tilde{S}_{-1i} = \langle \Xi|c_{-i}c_0c_1|0\rangle\,, \qquad \tilde{S}_{0i} = -\langle \Xi|c_{-i}c_{-1}c_1|0\rangle\,, \qquad \tilde{S}_{1i} = \langle \Xi|c_{-i}c_{-1}c_0|0\rangle\,. \tag{2.17}$$

To determine the correlation functions on the left-hand side of the above equations we require the knowledge of the following similarity transformations.

$$\begin{aligned} U_{W_\infty}\alpha^{\mu}_{m}U^{-1}_{W_\infty} &= \oint \frac{dz}{2\pi i} z^m W_\infty \circ \partial X(z) \\ &= \sum_{n=m}^{\infty} \mathcal{A}_{mn}\alpha^{\mu}_{n}\,, \end{aligned} \tag{2.18}$$

where

$$\begin{aligned} W_\infty &= \lim_{n\to\infty} \tan\left(\frac{2}{n}\tan^{-1} z\right) \\ &= \lim_{n\to\infty} \frac{2}{n}\tan^{-1} z\,. \end{aligned} \tag{2.19}$$

The correlation functions given in (2.14), (2.16) and (2.17) are invariant under the scaling $z \to nz/2$. Therefore it is sufficient to work with $W_\infty(z) = \tan^{-1} z$ for the purposes of evaluating the similarity transformations. Thus $\mathcal{A}_{mn}$ is given by the expansion

$$(\tan z)^m = \sum_{n=m}^{\infty} \mathcal{A}_{mn} z^n \,. \tag{2.20}$$

The similarity transformation for the ghosts are obtained from

$$\begin{aligned} U_{W_\infty} b_i U_{W_\infty}^{-1} &= \oint \frac{dz}{2\pi i} z^{i+1} W_\infty \circ b(z) \\ &= \sum_{j=i}^{\infty} \mathcal{B}_{ij} b_j \,. \end{aligned} \tag{2.21}$$

Here $\mathcal{B}_{ij}$ is defined in terms of the expansion

$$(\tan z)^{i+1} \cos^2 z = \sum_{j=i}^{\infty} \mathcal{B}_{ij} z^{j+1} \,. \tag{2.22}$$

Similarly the the matrix $\mathcal{C}$ is represents the transformation of the $c$ ghost by $U_{W_\infty}$.

$$\begin{aligned} U_{W_\infty} c_s U_{W_\infty}^{-1} &= \int \frac{dz}{2\pi i} z^{s-2} W_\infty \circ c(z) \\ &= \sum_{t=s}^{\infty} \mathcal{C}_{st} c_t \end{aligned} \tag{2.23}$$

Again $\mathcal{C}_{st}$ is defined in terms of the expansion

$$(\tan z)^{s-2} \cos^{-4} z = \sum_{s=t}^{\infty} \mathcal{C}_{st} z^{t-2} \,. \tag{2.24}$$

In appendix A we have listed down a few components of the matrices $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$. It is easy to see that from the definition of these matrices given above they are twist invariant. For example $\mathcal{A}_{mn} = 0$ if $m + n$ is an odd number.

Substituting the matrices corresponding to the similarity transformation in (2.14) we obtain

$$\begin{aligned} S_{mn} &= \frac{1}{mn} \sum_{m'=1}^{n} m' \mathcal{A}_{(-m)m'} \mathcal{A}_{(-n)(-m')} \left( \mathcal{C}_{(-1)(-1)} \mathcal{C}_{00} \mathcal{C}_{11} \right) \\ &= \frac{1}{mn} \sum_{m'=1}^{n} m' \mathcal{A}_{(-m)m'} \mathcal{A}_{(-n)(-m')} \,. \end{aligned} \tag{2.25}$$

Here we have used the fact that the matrices $\mathcal{A}$ and $\mathcal{C}$ are upper triangular. In the second line we have used $\mathcal{C}_{(-1)(-1)} = \mathcal{C}_{00} = \mathcal{C}_{11} = 1$ and $\langle c_{-1} c_0 c_1 \rangle = 1$. Since the matrix $\mathcal{A}$ is twist invariant the width matrix $S$ is also twist invariant. We write down a few components of the width matrix $S$. These can be evaluated by the coefficients of the $\mathcal{A}$ matrix listed in appendix A.[5]

$$\begin{aligned} S_{11} &= -\frac{1}{3}\,, \qquad S_{22} = -\frac{1}{30}\,, \qquad S_{13} = \frac{4}{45}\,, \\ S_{33} &= -\frac{83}{2835}\,, \qquad S_{24} = \frac{16}{945}\,, \qquad S_{44} = \frac{109}{11340}\,. \end{aligned} \tag{2.26}$$

For the width matrix of the ghost sector $\tilde{S}$ we have

$$\begin{aligned} &\tilde{S}_{-1i} = \mathcal{C}_{-i-1}\,, \qquad \tilde{S}_{0i} = \mathcal{C}_{(-i)0}\,, \\ &\tilde{S}_{1i} = \mathcal{C}_{(-i)1} - \mathcal{C}_{(-i)(-1)}\mathcal{C}_{(-1)1}\,, \qquad \tilde{S}_{si} = \sum_{k=-1}^{s} \mathcal{C}_{(-i)k}\mathcal{B}_{(-s)(-k)}\,. \end{aligned} \tag{2.27}$$

Here $i \geq 2$ and $s \geq 2$. Using the above equations and the coefficients of the $\mathcal{C}$ and $\mathcal{B}$ matrices listed in appendix A, we find the following coefficients

$$\begin{aligned} &\tilde{S}_{(-1)3} = \frac{1}{3}\,, \qquad \tilde{S}_{(-1)5} = -\frac{7}{45}\,, \qquad \tilde{S}_{02} = \frac{2}{3}\,, \qquad \tilde{S}_{04} = -\frac{2}{15}\,, \qquad \tilde{S}_{13} = -\frac{1}{3}\,, \\ &\tilde{S}_{15} = -\frac{7}{45}\,, \qquad \tilde{S}_{22} = -\frac{29}{45}\,, \qquad \tilde{S}_{24} = \frac{128}{945}\,, \qquad \tilde{S}_{33} = \frac{61}{189}\,, \\ &\tilde{S}_{35} = -\frac{2176}{14175}\,. \end{aligned} \tag{2.28}$$

The sliver state can also be constructed using the explicit representation of the Witten vertex in terms of the Neumann functions [37, 38, 39] as proposed in [18]. These width coefficients obtained above are useful in comparing with this explicit construction of the sliver state. The matter sector width matrix $S$ has been compared to the explicit construction of the matter sector of the sliver state in [31]. The difficulty in comparing the ghost sector is that the representation of the Witten vertex with the Neumann functions is on the ghost number two vacuum. This makes the evaluation of the star product difficult for a ghost number zero state like the sliver state. It is of interest to point out that the ghost sector of the squeezed state constructed in [18] satisfies the Feynman-Siegel gauge and it has ghost number one, unlike the sliver state which does not satisfy the Feynman-Siegel gauge condition and has ghost number zero.

## 3. The generalized gluing and resmoothing theorem

It is clear that the sliver state consists of infinite number of levels above the SL$(2,\mathbb{R})$ vacuum. It would be difficult to compute the Witten vertex of excitations on the sliver state directly. A direct computation of the Witten vertex of three ghost number one states on the sliver using the Neumann function representation of the Witten

[5] Note that one can obtain the coefficients of the width matrix listed in [31] from these coefficients by replacing $\alpha^\mu_m$ by $i\sqrt{m}\alpha^\mu_m$. This is the normalization of the matter oscillators used in [31].

vertex leads to a determinant of a matrix of infinite dimension. It is therefore useful to develop a convenient method to compute the Witten vertex and the quadratic term. The generalized gluing and resmoothing theorem [40, 41] provides a means to evaluate these vertices. In this section we will review the algebraic statement of the theorem. Then we will apply it to re-derive the product law of the wedge states.

### 3.1 Statement of the theorem

We will follow [41] in the algebraic statement of the theorem and its application. Consider a set of conformal maps $f_1, \ldots, f_n, g_1, \ldots, g_m, f$ and $g$ with the property that they are well defined and one-to-one inside a disc around the origin. Let $f$ and $g$ leave the origin fixed. The conformal field theory of our interest is defined in the upper half plane. Therefore the functions $f_i, g_i, f, g$ map the real axis to the real axis. The generalized gluing and resmoothing theorem states that

$$\begin{aligned}\sum_r \langle f_1 \circ \Phi_{r1}(0) \cdots f_n \circ \Phi_{r_n}(0) f \circ \Phi_r(0)\rangle \langle g_1 \circ \Phi_{s1}(0) \cdots g_m \circ \Phi_{s_m}(0) g \circ \Phi_r^c(0)\rangle = \\ = e^{cK} \langle F \circ f_1 \circ \Phi_{r1}(0) \cdots F \circ f_n \circ \Phi_{r_n}(0) G \circ I \circ g_1 \circ \Phi_{s_1}(0) \cdots \\ \cdots G \circ I \circ g_m \circ \Phi_{s_m}(0)\rangle \,. \end{aligned} \tag{3.1}$$

Here $\{|\Phi_r\rangle\}$ is a complete set of basis and $\{\langle \Phi_r^c|\}$ is basis dual to it with the property

$$\langle \Phi_r^c | \Phi_s \rangle = \delta_{rs} \tag{3.2}$$

$F$ and $G$ are two conformal maps with the following properties.

- $F$ is a conformal map which is well defined and one-to-one outside a disc around the origin. It leaves leaves the point at infinity fixed.
- $G$ is a conformal map which is well defined and one-to-one around the origin ($z = 0$). It leaves the origin fixed.
- The maps $F \circ f(z)$ and $G \circ I \circ g \circ I(z)$ are well defined over an annulus around the origin in the $z$ plane and in this region
$$F \circ f(z) = G \circ I \circ g \circ I(z)\,. \tag{3.3}$$
- Let the region in the unit disc $|z| \leq 1$ be mapped to the region $D_1^c$ under the map $f(z)(\equiv u)$. Call the complement of this region $D_1$. Similarly let the region outside the unit disc $|z| \geq 1$ be mapped to the region $D_2^c$ under the map $g(z) \circ I(z)(\equiv v)$. Call the complement of this region $D_2$. The image of $D_1$ under the map $F(u)(\equiv w)$ and the image of $D_2$ under the map $G \circ I(v)(\equiv w)$ are complements of each other in the $w$ plane. The maps $F$ and $G \circ I$ are well defined in regions $D_1$ and $D_2$ (See figure 1 for the various domains and ranges involved.)

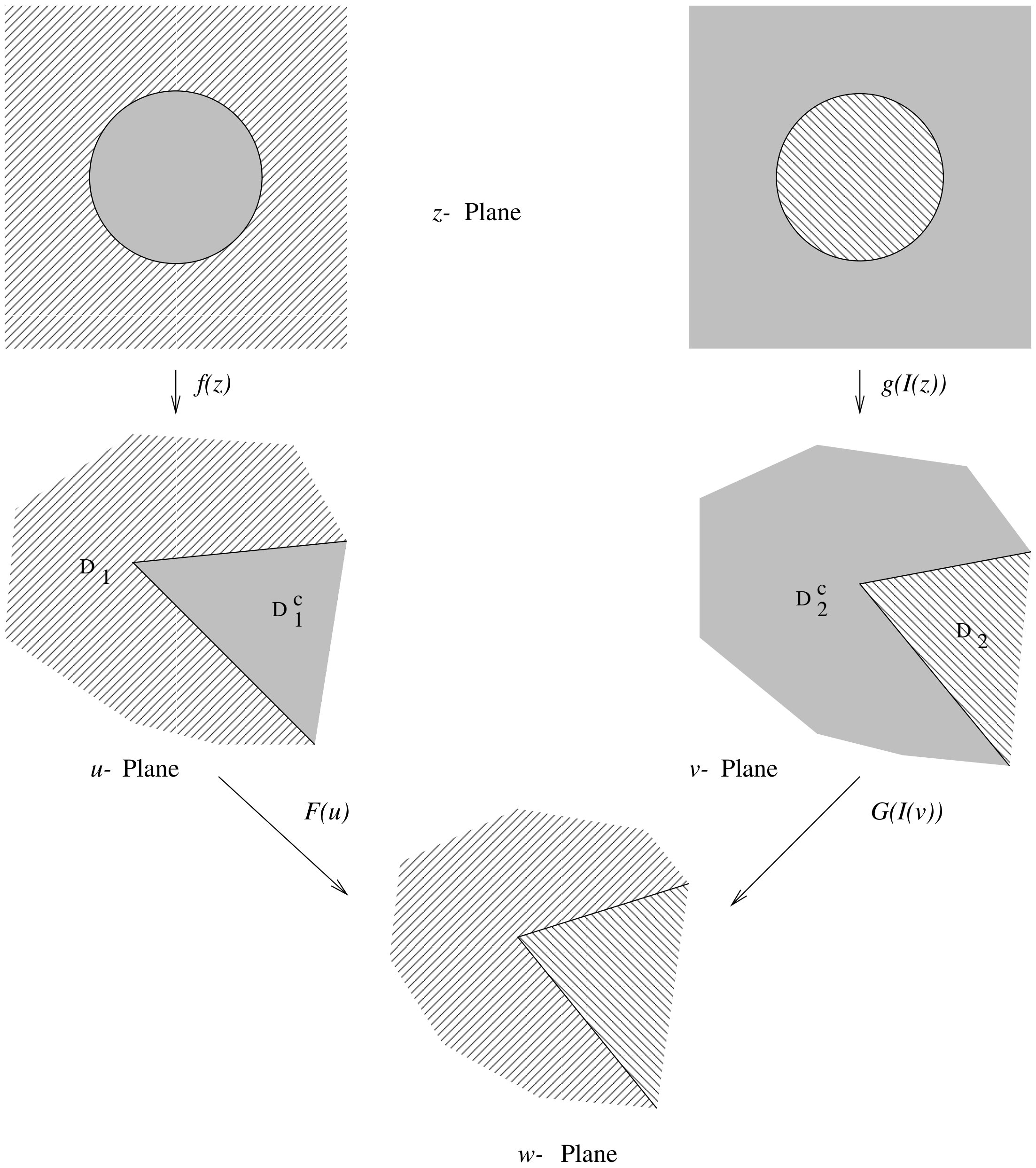


**Figure 1:** Domains in the generalized gluing and resmoothing theorem

The factor $e^{cK}$ in (3.1) depends on the maps $f$ and $g$ and the central charge $c$ of the theory. For the critical bosonic string theory in $d = 26$ the central charge of the ghost conformal field theory and the matter conformal field theory is zero. Thus this factor becomes unity for the theory of our interest.

### 3.2 Product law of wedge states

In this section we will derive the product law of the wedge states found in [13]. It was shown using using the geometrical description of the generalized gluing and resmoothing theorem that $|n\rangle * |m\rangle = |n+m-1\rangle$. We will re-derive this using the algebraic form of the generalized gluing and resmoothing theorem. This will also help in developing the method of evaluating the Witten vertex for excitations above the sliver state.

We have seen that the wedge state is given by

$$|n\rangle = U^{\dagger}_{W_n}|0\rangle = U_{I\circ W_n^{-}\circ I}|0\rangle\,. \tag{3.4}$$

Inserting a complete set of basis $\sum_r |\Phi_r\rangle\langle\Phi^c_r|$ we get

$$\begin{aligned} |n\rangle &= \sum_r |\Phi_r\rangle\langle 0|I\circ\Phi^c_r(0)U_{I\circ W_n^{-1}\circ I}|0\rangle \\ &= \sum_r |\Phi_r\rangle\langle 0|U_{I\circ W_n\circ I}\, I\circ\Phi^c_r(0)U_{I\circ W_n^{-1}\circ I}|0\rangle \\ &= \sum_r |\Phi_r\rangle\langle 0|W_n\circ\Phi^c_r(0)|0\rangle\,. \end{aligned} \tag{3.5}$$

Here we have used $\langle 0|U_{I\circ W_n\circ I} = \langle 0|$, this can be seen from (2.11). In the third line we have used the definition of conformal transformation on the field $\Phi^c_r(0)$ and the fact that correlation functions are invariant under $\mathrm{SL}(2,\mathbb{R})$ transformation. Using (3.5) to express the wedge state $|n\rangle$ and $|m\rangle$ in terms of a complete set of basis we can write the star product of these states with an arbitrary state $|\Phi\rangle$ as

$$\langle V_3|\;|\Phi\rangle\otimes|n\rangle\otimes|m\rangle = \sum_{r,s}\langle f_1\circ\Phi(0)f_2\circ\Phi_r(0)f_3\circ\Phi_s(0)\rangle\langle W_n\circ\Phi^c_r(0)\rangle\langle W_m\circ\Phi^c_s(0)\rangle\,. \tag{3.6}$$

Here $\langle V_3|$ stands for the Witten vertex. The functions $f_1, f_2$ and $f_3$ are given by

$$\begin{aligned} f_1 &= M\left(e^{2\pi i/3}\left[\frac{1+iz}{1-iz}\right]^{2/3}\right), \qquad f_2 = M\left(\left[\frac{1+iz}{1-iz}\right]^{2/3}\right), \\ f_3 &= M\left(e^{4\pi i/3}\left[\frac{1+iz}{1-iz}\right]^{2/3}\right). \end{aligned} \tag{3.7}$$

where $M$ stands for the $\mathrm{SL}(2,\mathbb{C})$ transformation which maps the unit circle to the real axis given in (2.3).

Now we use the generalized gluing and resmoothing theorem to sum over $r$ and $s$. As $f_2(0) = 0$ and $W_n(0) = 0$ we will first sum over $r$. From the statement of the theorem, we need to find the functions $F_1$ and $G_1$ such that

$$F_1\circ f_2(z) = G_1\circ I\circ W_n\circ I(z)\,. \tag{3.8}$$

We look for these functions with the following ansatz

$$F_1(u) = M(e^{i\phi_1}(M^{-1}(u)^{\alpha_1}))\,, \qquad G_1\circ I(v) = M((M^{-1}(v)^{\beta_1}))\,. \tag{3.9}$$

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The condition in (3.8) gives

$$M\left[e^{i\phi_1}\left(\frac{1+iz}{1-iz}\right)^{2\alpha_1/3}\right]=M\left[\left(\frac{1-i/z}{1+i/z}\right)^{2\beta_1/n}\right]. \tag{3.10}$$

By comparing both sides we must have

$$\frac{2\alpha_1}{3}=\frac{2\beta_1}{n}. \tag{3.11}$$

The map $(1+iz/1-iz)^{2/3}=(\hat{u})^{\alpha_1}$ takes the region inside disc $|z|\leq 1$ to a wedge of angle $2\pi/3$. Let $D_1$ be the complement of this region. $D_1$ has a wedge angle of $4\pi/3$ and is bounded by straight lines passing thorough the origin at angles $\pi/3$ and $5\pi/3$. Now $e^{i\phi_1}\hat{u}^{\alpha_1}$ scales the wedge angle of the complement region to $4\pi\alpha_1/3$. The phase rotates the region so that now it is bounded by straight lines passing through $\pi\alpha_1/3+\phi_1$ and $5\pi\alpha_1/3+\phi_1$ Call this region wedge $E_1$. Similarly the map $(1-i/z)/(1+i/z)^{2/n}=\hat{v}$ takes the outside of the unit disc $|z|\geq 1$ to a wedge of angle $2\pi/n$. The complement of this region is the wedge of angle $2\pi(n-1)/n$. Let this region be $D_2$ Under the scaling $\hat{v}^{\beta_1}=w$ the region $D_2$ is mapped to a region $E_2$. This is a wedge of angle $2\pi\beta_1(n-1)/n$. The condition for the maps $F_1$ and $G_1$ is that $E_1$ and $E_2$ should be the complements of each other in the $w$ plane. This gives

$$\frac{4\pi\alpha_1}{3}+2\pi\frac{n-1}{n}\beta_1=2\pi\,. \tag{3.12}$$

Matching the phase angles so that the wedges $E_1$ and $E_2$ are complements of each other we get

$$\frac{\pi\alpha_1}{3}+\phi_1=\left(\frac{\pi}{n}+\frac{2\pi(n-1)}{n}\right)\beta_1\,. \tag{3.13}$$

The solution of (3.11), (3.12) and (3.13) are given by

$$\alpha_1=\frac{3}{n+1}\,,\qquad \beta_1=\frac{n}{n+1}\,,\qquad \phi_1=\frac{2\pi(n-1)}{n+1}\,. \tag{3.14}$$

Therefore by applying the generalized gluing and resmoothing theorem for summing over $r$ in (3.6) we obtain

$$\langle V_3|\,|\Phi\rangle\otimes|n\rangle\otimes|m\rangle=\langle F_1\circ f_2\circ\Phi(0)F_1\circ f_3\circ\Phi_s(0)\rangle\langle W_m\circ\Phi_s^c(0)\rangle\,. \tag{3.15}$$

Using the definition of $F_1$ we find

$$\begin{aligned} F_1\circ f_2(z)&=M\left[e^{\frac{2\pi in}{n+1}}\left(\frac{1+iz}{1-iz}\right)^{\frac{2}{n+1}}\right],\\ F_1\circ f_3(z)&=M\left[\left(\frac{1+iz}{1-iz}\right)^{\frac{2}{n+1}}\right] \end{aligned} \tag{3.16}$$

We use the same method to sum over $s$. We look for functions $F_2$ and $G_2$ such that

$$F_2 \circ F_1 \circ f_3(z) = G_2 \circ I \circ W_m \circ I(z) \,. \tag{3.17}$$

We make the following antsaz for $F_2$ and $G_2$

$$F_2(u) = M(e^{i\phi_2}(M^{-1}(u))^{\alpha_2}) \,, \qquad G_2 \circ I(v) = M((M^{-1}(v))^{\beta_2}) \,. \tag{3.18}$$

Using the similar methods of matching domains in the $w$-plane we obtain the equations for $\alpha_2, \beta_2, \phi_2$,

$$\begin{gathered} \frac{2\alpha_2}{n+1} = \frac{2\beta_2}{m} \,, \qquad \frac{n}{n+1} 2\pi\alpha_2 + \frac{m-1}{m} 2\pi\beta_2 = 2\pi \,, \\ \frac{\pi}{n+1}\alpha_2 + \phi_2 = \left( \frac{\pi}{m} + 2\pi \frac{m-1}{m} \right) \beta_2 \,. \end{gathered} \tag{3.19}$$

The solutions of the above set of equations are given by

$$\alpha_2 = \frac{n+1}{n+m-1} \,, \qquad \beta_2 = \frac{m}{n+m-1} \,, \qquad \phi_2 = \frac{2\pi(m-1)}{n+m-1} \,. \tag{3.20}$$

The definition of $F_2$ allows us to find

$$\begin{aligned} F_2 \circ F_1 \circ f_1(z) &= M \left[ \left( \frac{1+iz}{1-iz} \right)^{\frac{2}{n+m-1}} \right] \\ &= W_{n+m-1}(z) \,. \end{aligned} \tag{3.21}$$

We can now use the generalized gluing and resmoothing theorem to sum over $s$ in (3.15). We obtain

$$\langle V_3 || \Phi \rangle \otimes |n\rangle \otimes |m\rangle = \langle W_{n+m-1} \circ \Phi(0) \rangle \,. \tag{3.22}$$

We have taken $\Phi$ to be any string state. $W_{n+m-1}$ is the conformal transformation associated with the wedge state $|n+m-1\rangle$. Thus we have proved

$$|n\rangle * |m\rangle = |n+m-1\rangle \,. \tag{3.23}$$

## 4. Excitations on wedge states

In this section we define the excitations that we will consider acting on the sliver state. We will then use the generalized gluing and resmoothing theorem to evaluate the Witten vertex and the kinetic term for string field theory actions around the tachyon vacuum.

### 4.1 The similarity transformation

As we have seen in the section 2, the wedge states are defined by

$$\langle n| = \langle 0|U_{W_n}\,, \qquad |n\rangle = U_{I\circ W_n^{-1}\circ I}|0\rangle\,. \tag{4.1}$$

$U_{W_n}$ has definite representation in terms of generators of the Virasoro group. The wedge states are thus defined as a conformal transformation acting on the SL$(2,\mathbb{R})$ vacuum. To obtain creation and annihilation operators on the wedge state $|n\rangle$ it is natural to define operators using the similarity transformation given below

$$\mathcal{O}^{(n)} = U_{I\circ W_n^{-1}\circ I}\mathcal{O}U_{I\circ W_n\circ I}\,, \tag{4.2}$$

which implies

$$\mathcal{O}^{(n)}|n\rangle = U_{I\circ W_n^{-1}\circ I}\mathcal{O}|0\rangle\,. \tag{4.3}$$

Using this definition, for example the operator $c^{(n)}_{-m}$ for $m \geq -1$ act as creation operators on the state $|n\rangle$. for $m < -1$ the operators $c^{(n)}_{-m}$ annihilate the state $|n\rangle$. The transformed operators give a convenient way of organizing the operators as creation and annihilation operators over the wedge states. We will denote $\mathcal{O}^{(\infty)}$ as $\tilde{\mathcal{O}}$. Thus

$$\tilde{\mathcal{O}} = U_{I\circ W_\infty^{-1}\circ I}\mathcal{O}U_{I\circ W_\infty\circ I}\,. \tag{4.4}$$

On the bra $\langle n|$ the similarity transformation for operators is the BPZ conjugate of (4.2). It is given by

$$\left(\mathcal{O}^{(n)}\right)^\dagger = U_{W_n}^{-1}\mathcal{O}^\dagger U_{W_n}\,, \tag{4.5}$$

similarly

$$\tilde{\mathcal{O}}^\dagger = U_{W_\infty}^{-1}\mathcal{O}^\dagger U_{W_\infty}\,. \tag{4.6}$$

It is interesting to point out that the conservation laws of the wedge state $\langle n|$ obtained in [13, 31] is due to the similarity transformation of annihilation operators acting on the state $\langle n|$. We demonstrate it for the sliver state. It is clear form the definition of $\tilde{c}_m$ that

$$\langle\Xi|\tilde{c}^\dagger_m = 0\,, \qquad \text{for } m \leq -2\,. \tag{4.7}$$

Now let us express $\tilde{c}^\dagger_m$ in terms of the conventional $c$'s. Using the definition of $\tilde{c}^\dagger_m$ we have

$$\begin{aligned}
\tilde{c}^\dagger_m &= U_{W_\infty}^{-1}c_m U_{W_\infty} \\
&= \oint \frac{dz}{2\pi i} z^{m-2} W_\infty^{-1}\circ c(z) \\
&= \oint \frac{dz}{2\pi i}\frac{1}{(1+z^2)^2}(\tan^{-1}z)^{m-2}c(z)\,.
\end{aligned} \tag{4.8}$$

It is sufficient to use $W_\infty^{-1}(z) = \tan(z)$. We have changed the variable of integration in the third line. Evaluating the integral for the cases $m = -2, -3, -4, -5, -6$ gives

$$\begin{aligned}
\tilde{c}^{\dagger}_{-2} &= c_{-2} - \frac{2}{3}c_0 + \frac{29}{45}c_2 - \frac{608}{945}c_4 + \cdots \,, \\
\tilde{c}^{\dagger}_{-3} &= c_{-3} - \frac{1}{3}c_{-1} + \frac{1}{3}c_1 - \frac{61}{189}c_3 + \cdots \,, \\
\tilde{c}^{\dagger}_{-4} &= c_{-4} + \frac{2}{15}c_0 - \frac{128}{945}c_2 + \frac{629}{4725}c_4 + \cdots \,, \\
\tilde{c}^{\dagger}_{-5} &= c_{-5} + \frac{7}{45}c_{-1} - \frac{7}{45}c_1 + \frac{2176}{14175}c_3 + \cdots \,, \\
\tilde{c}^{\dagger}_{-2} &= c_{-6} - \frac{2}{35}c_0 + \frac{848}{14175}c_2 - \frac{1312}{22275}c_4 + \cdots \,.
\end{aligned} \tag{4.9}$$

It is clear that these are the conservation laws obtained for the sliver state in [31].

### 4.2 The Witten vertex for transformed operators

In this section we derive a general formula for computing the Witten vertex of excited states defined in the previous section. To be definite we evaluate

$$S^3_{pnm} = \langle V_3|\, \Omega^{(p)}(0)|p\rangle \otimes \Psi^{(n)}(0)|n\rangle \otimes \Lambda^{(m)}(0)|m\rangle \,. \tag{4.10}$$

We write the excited state $|\Psi^{(n)}\rangle$ in terms of a complete set of basis as follows.

$$\begin{aligned}
\Psi^{(n)}|n\rangle &= U_{I\circ W_n^{-1}\circ I}\Psi(0)|0\rangle \\
&= \sum_r |\Phi_r\rangle\langle 0|I\circ\Phi^c_r(0)U_{I\circ W_n^{-1}\circ I}\Psi(0)|0\rangle \\
&= \sum_r |\Phi_r\rangle\langle 0|U_{I\circ W_n\circ I} I\circ\Phi^c_r(0)U_{I\circ W_n^{-1}\circ I}\Psi(0)|0\rangle \\
&= \sum_r |\Phi_r\rangle\langle I\circ\Psi(0)W_n\circ\Phi^c_r(0)\rangle \,.
\end{aligned} \tag{4.11}$$

In the last step we have used the fact that correlation functions are invariant under $\mathrm{SL}(2,\mathbb{R})$ transformation $I(z)$. Using a similar expansion in terms of a complete set of basis for the states $\Omega^{(p)}|p\rangle$ and $\Lambda^{(m)}|m\rangle$ we obtain the following formula for the Witten vertex $S^3_{pnm}$

$$\begin{aligned}
S^3_{pnm} = \sum_{r,s,t} \big(&\langle f_1\circ\Phi_t(0) f_2\circ\Phi_r(0) f_3\circ\Phi_s(0)\rangle \times \\
&\times \langle I\circ\Omega(0)W_p\circ\Phi^c_t(0)\rangle\langle I\circ\Psi(0)W_n\circ\Phi^c_r(0)\rangle\langle I\circ\Lambda(0)W_m\circ\Phi^c_s(0)\rangle\big) \,.
\end{aligned} \tag{4.12}$$

Summing over $r$ and $s$ using the generalized gluing and resmoothing theorem as stated in (3.1) twice we get

$$S^3_{pnm} = \sum_t \langle F_2\circ F_1\circ f_1\circ\Phi_t(0) F_2\circ G_1\circ\Psi(0) G_2\circ\Lambda(0)\rangle\langle I\circ\Omega(0)W_p\circ\Phi^c_t(0)\rangle \,. \tag{4.13}$$

In the above equation the functions $F_1, G_1$ and $F_2, G_2$ are as found in (3.9) and (3.18), respectively. We now use the generalized gluing and resmoothing theorem again to sum over t. For this we need to find functions $F_3$ and $G_3$ which satisfy

$$F_3 \circ F_2 \circ F_1 \circ f_1(z) = F_3 \circ I \circ W_p \circ I(z)\,. \tag{4.14}$$

We again make the antsaz

$$F_3(u) = M(e^{i\phi_3}(M^{-1}(u))^{\alpha_3})\,, \qquad G_3 \circ I(v) = M((M^{-1}(v))^{\beta_3})\,. \tag{4.15}$$

Using this antsaz and the methods described in the previous section we obtain the following equations for $\alpha_3, \beta_3, \phi_3$.

$$\begin{aligned} \frac{2\alpha_3}{n+m-1} &= \frac{2\beta_3}{p}\,, \qquad \left(\frac{n+m-2}{n+m-1}\right) 2\pi\alpha_3 + \left(\frac{p-1}{p}\right) 2\pi\beta_3 = 2\pi\,, \\ \frac{\pi\alpha_3}{n+m-1} + \phi_3 &= \left(\frac{\pi}{p} + \frac{2\pi(p-1)}{p}\right)\beta_3\,. \end{aligned} \tag{4.16}$$

The solution of these set of equations is given by

$$\alpha_3 = \frac{n+m-1}{n+m+p-3}\,, \qquad \beta_3 = \frac{p}{n+m+p-3}\,, \qquad \phi_3 = \frac{2\pi(p-1)}{n+m+p-3}\,. \tag{4.17}$$

Now summing over $t$ in (4.13) using the generalized gluing and resmoothing theorem we get

$$S^3_{pnm} = \langle G_3 \circ \Omega(0) F_3 \circ F_2 \circ G_1 \circ \Psi(0) F_3 \circ G_2 \circ \Lambda(0) \rangle\,, \tag{4.18}$$

where

$$\begin{aligned} F_3 \circ F_2 \circ G_1 \circ (z) &= M\left[ e^{\frac{i\pi n + 2\pi i(n+p-2)}{n+m+p-3}} \left(\frac{1+iz}{1-iz}\right)^{\frac{n}{n+m+p-3}} \right], \\ F_3 \circ G_2(z) &= M\left[ e^{\frac{i\pi m + 2\pi i(p-1)}{n+m+p-3}} \left(\frac{1+iz}{1-iz}\right)^{\frac{m}{n+m+p-3}} \right], \\ G_3(z) &= M\left[ e^{\frac{i\pi p}{n+m+p-3}} \left(\frac{1+iz}{1-iz}\right)^{\frac{p}{n+m+p-3}} \right]. \end{aligned} \tag{4.19}$$

We have thus obtained the Witten vertex for excitations on wedge states. Let us perform some simple checks on the vertex we have obtained. As the wedge state $|2\rangle$ is the vacuum state we expect the vertex $S^3_{222}$ to be the ordinary Witten vertex. This can be easily checked from (4.19). We see that (4.19) reduces to the ordinary Witten vertex for $n = m = p = 2$. The wedge state $|1\rangle$ is the identity of the string algebra. This is not a normalizable state. This explains the reason that the vertex $S^3_{111}$ is not well defined. We note that the vertex $S^3_{nnn}$ for any $n$ has a cyclic symmetry in the fields $\Omega(0), \Phi(0)$ and $\Lambda(0)$. This can be easily seen by the $\mathrm{SL}(2,\mathbb{R})$ transformation $M \circ \exp((2\pi i/3) \circ M^{-1}$ which permutes these fields. This symmetry is different from the cyclic symmetry of the vertex in the fields $\Omega^{(p)}(0), \Phi^{(n)}(0), \Lambda^{(m)}(0)$. The latter symmetry is due to the cyclic symmetry of the Witten vertex.

### 4.3 The quadratic term for the transformed operators

In this section we will calculate the quadratic term for the transformed operators. Let the BRST operator at the tachyon vacuum be $Q$. The quadratic term in the string field theory action is given by

$$S^2 = \langle I \circ \Phi(0) Q \Phi(0) \rangle \,. \tag{4.20}$$

Let us now evaluate the kinetic term for states $\Lambda^{(m)}(0)|m\rangle$ and $\Psi^{(n)}(0)|n\rangle$. Substituting these states in the kinetic term we get

$$\begin{aligned} S^2_{mn} &= \langle I \circ \Lambda(0) U_{W_m} Q U_{I \circ W_n^{-1} \circ I} \Phi(0) \rangle \\ &= \langle I \circ \Lambda(0) U_{W_m} U_{I \circ W_n^{-1} \circ I} \tilde{Q} \Phi(0) \rangle \,, \end{aligned} \tag{4.21}$$

where $\tilde{Q} = U_{I \circ W_n \circ I} Q U_{I \circ W_n^{-1} \circ I}$. Inserting a complete set of states in the above correlation function we get

$$S^2_{nm} = \sum_r \langle I \circ \Lambda(0) W_n \circ \Phi_r(0) \rangle \langle W_m \circ \Phi^c_r(0) I \circ (\tilde{Q} \Phi(0)) \rangle \,. \tag{4.22}$$

We will use the generalized gluing and resmoothing theorem to sum over $r$. We need to find functions $F'$ and $G'$ such that

$$F' \circ W_n(z) = G' \circ I \circ W_m \circ I(z) \,. \tag{4.23}$$

We make the usual anstaz

$$F'(u) = M(e^{i\phi'} (M^{-1}(u))^{\alpha'}) \,, \qquad G'(v) = M((M^{-1}(v))^{\beta'}) \,. \tag{4.24}$$

Using the methods discussed in the previous section we obtain the following equations for $\alpha', \beta'$ and $\phi'$

$$\begin{aligned} \frac{2\alpha'}{n} &= \frac{2\beta'}{m} \,, \qquad 2\pi\alpha' \left( \frac{n-1}{n} \right) + 2\pi\beta' \left( \frac{m-1}{m} \right) = 2\pi \,, \\ \frac{\pi\alpha'_2}{n} + \phi' &= \left( 2\pi \frac{m-1}{m} + \frac{\pi}{m} \right) \beta' \,. \end{aligned} \tag{4.25}$$

The solutions are given by

$$\alpha' = \frac{n}{n+m-2} \,, \qquad \beta' = \frac{m}{n+m-2} \,, \qquad \phi' = 2\pi \frac{m-1}{n+m-2} \,. \tag{4.26}$$

So after summing over $s$ we obtain

$$S^2_{nm} = \langle F' \circ I \circ \Lambda(0) G' \circ \tilde{Q} G' \circ \Phi(0) \rangle \,. \tag{4.27}$$



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The functions in the vertex $S^2_{mn}$ is given by

$$G'(z) = M\left[e^{\frac{i\pi m}{n+m-2}}\left(\frac{1+iz}{1-iz}\right)^{\frac{m}{n+m-2}}\right],$$
$$F'\circ I(z) = M\left[e^{\frac{i\pi n+2\pi i(m-1)}{n+m-2}}\left(\frac{1+iz}{1-iz}\right)^{\frac{n}{n+m-2}}\right]. \qquad (4.28)$$

Note that the vertex $S^2_{mn}$ reduces to the ordinary kinetic term given in (4.20) for $m = n = 2$ as expected. The quadratic term $S^2_{11}$ is not well defined due to the fact that the wedge state $|1\rangle$ is not normalizable.

## 5. Effective action for excitations on the sliver

We have evaluated the Witten vertex and the quadratic term for excitations on wedge states. In this section we will derive the Witten vertex and the quadratic term for excitations on the sliver state by taking an appropriate limit of the vertices derived in section 4. We then discuss the discrete symmetries for the quadratic term and the Witten vertex for excitations above the sliver. The effective action is explicitly computed for level two excitations on the sliver. Then we analyze the local maxima of the action.

### 5.1 The sliver limit for vertices

Consider the Witten vertex $S^3_{pnm}$, naively one would expect the the vertex for excitations on the sliver is obtained by taking the limit $p, n, m \to \infty$ in $S^3_{pnm}$. The resulting expression is not well defined. This can be easily seen from (4.19). However the limit $n \to \infty$ in $S^3_{nnn}$ is well defined. As discussed at the end of section 4.2 this method of defining the limit also preserves the cyclic symmetry of the vertex for the fields $\Omega(0), \Phi(0), \Lambda(0)$. This requirement of cyclic symmetry in these fields is consistent with the expectation that the translational invariant sliver state is unique. We encounter a similar situation with the quadratic term. The naive limit $m, n \to \infty$ in $S^2_{mn}$ is not well defined. But, the limit $n \to \infty$ in $S^2_{nn}$ is well defined. It preserves the exchange symmetry of the quadratic term for fields $\Phi(0)$ and $\Lambda(0)$ which represent excitations on the unique sliver state.

#### 5.1.1 The quadratic term

As we have discussed above the sliver limit for the quadratic term is obtain by

$$\begin{aligned}\tilde{S}^2 &= \lim_{n\to\infty} S^2_{nn}\\ &= \langle g_1\circ\Phi(0) g_2\circ\tilde{Q} g_2\circ\Lambda(0)\rangle\,. \end{aligned}\qquad (5.1)$$

After performing the $\mathrm{SL}(2,\mathbb{R})$ transformation $M \circ \exp(-i\pi/2) \circ M^{-1}$ on the vertex $\tilde{S}^2$ the functions $g_1(z), g_2(z)$ are given by

$$\begin{aligned}
g_1(z) &= M\left[e^{i\pi}\left(\frac{1+iz}{1-iz}\right)^{1/2}\right] \\
&= -\frac{2}{z} - \frac{1}{2}z + \frac{1}{8}z^3 - \frac{1}{8}z^5 + \cdots, \\
g_2(z) &= M\left[\left(\frac{1+iz}{1-iz}\right)^{1/2}\right] \\
&= \frac{1}{2}z - \frac{1}{8}z^3 + \frac{1}{16}z^5 - \frac{5}{128}z^7 \cdots .
\end{aligned} \tag{5.2}$$

We now define $\tilde{Q}$ in the sliver limit.

$$\tilde{Q} = \lim_{n\to\infty} U_{I\circ W_n \circ I} \circ Q U_{I\circ W_n^{-1}\circ I} \,. \tag{5.3}$$

In the sliver limit we can replace the operator $U_{W_n}$ by $U_{W_\infty}$ where $W_\infty(z) = \tan^{-1}(z)$ [13]. The operator $Q$ is hermitean, therefore $I \circ Q = -Q$. Now we will discuss the specific cases for $Q = Q_{\mathrm{BRST}}, Q = c_0$ and $Q = c_0 + (c_2 + c_{-2})/2$.

**Case 1.** $Q = Q_{\mathrm{BRST}}$
As the BRST current is a primary with weight one it commutes with the operator $U_{W_\infty}$. Therefore we get

$$\tilde{Q} = Q_{\mathrm{BRST}} \,. \tag{5.4}$$

Furthermore we also have the result $g_2 \circ Q_{\mathrm{BRST}} = Q_{\mathrm{BRST}}$.

**Case 2.** $Q = c_0$
Using the definition of $c_0$ and of the

$$\begin{aligned}
\tilde{Q} &= -I \circ \int \frac{dz}{2\pi i} z^{-2} W_\infty \circ c(z) \\
&= \sum_{t=0}^{\infty} \mathcal{C}_{0t} c_{-t} \,,
\end{aligned} \tag{5.5}$$

where the coefficients $\mathcal{C}_{0t}$ are defined in (2.24). We have use the fact that the $\mathcal{C}_{0t}$ is twist invariant and $I \circ c_{2k} = -c_{-2k}$. We now need to evaluate $g_2 \circ \tilde{Q}$. Writing the contour integral representation for $c_t$'s we get

$$g_2 \circ \tilde{Q} = \sum_{t=0}^{\infty} \mathcal{C}_{0t} \oint \frac{dz}{2\pi i} \frac{g_2 \circ c(z)}{z^{t+2}} \,. \tag{5.6}$$

Here the contour is around the origin $z = 0$. Now we perform the conformal transformation and change the variable of integration from $z \to \tan(2\tan^{-1} z)$. We obtain the following expression for $g_2 \circ \tilde{Q}$

$$g_2 \circ \tilde{Q} = \sum_{t=0}^{\infty} \mathcal{C}_{0t} \oint \frac{dz}{2\pi i} \left[\frac{1}{\tan(2\tan^{-1} z)}\right]^{t+2} \frac{4c(z)}{(1+z^2)^2 \cos^4(2\tan^{-1} z)}$$
$$= \sum_{t=0}^{\infty} \mathcal{C}_{Ot} \oint \frac{dz}{2\pi i} \mathcal{V}_{c_0}(z)\,, \tag{5.7}$$

where

$$\mathcal{V}_{c_0}(z) = \sum_{t=0}^{\infty} \mathcal{C}_{0t} \left[\frac{1}{\tan(2\tan^{-1} z)}\right]^{t+2} \frac{4c(z)}{(1+z^2)^2 \cos^4(2\tan^{-1} z)}\,. \tag{5.8}$$

We have to be careful about the contour in (5.7). The contour splits into two contours. One is counter clockwise around the origin and the other is counter clockwise around infinity. This is because both $\tan(2\tan^{-1} 0)$ and $\tan(2\tan^{-1}\infty)$ is zero. We now specify the prescription for choosing the contour. The correlation function $\tilde{S}^2$ for any two fields before performing the contour integral is a function of $z$. This function is defined below.

$$C(z) = \langle g_1 \circ \Phi(0) c(z) g_2 \circ \Lambda(0)\rangle$$
$$= \sum_n C_n z^n\,. \tag{5.9}$$

In the second line we have written down the Laurent series expansion of $C(z)$. Substituting this expansion in (5.7) we get

$$g_2 \circ \tilde{Q} = \sum_{t=0}^{\infty} \oint \frac{dz}{2\pi i} \left[\frac{1}{\tan(2\tan^{-1} z)}\right]^{t+2} \frac{4}{(1+z^2)^2 \cos^4(2\tan^{-1} z)} \times$$
$$\times \left(\sum_n C_n z^n\right). \tag{5.10}$$

It is easy to show that for the linear term and the terms in the above series with $t+1 \geq n$ and $t < -n$ and $n = 1, 2, \ldots$, the contour at infinity and the origin give the same result. So we have to detail the prescription for $t+1 < n$ and $t > -n$ with $n = 1, 2, \ldots$. The prescription we adopt is for terms with positive powers of $z$ the integral is chosen around the origin. For terms with negative powers of $z$ the contour is at infinity. This definition of the contour defines a BRST operator of the type $\sum_{n=0}^{\infty} a_{2n} \mathcal{Q}_{2n}$, where $\mathcal{Q}_{2n} = (c_{2n} + c_{-2n})$ and $a_n$ are constants. This BRST operator is hermitean and and satisfies the required properties to maintain gauge invariance of the cubic action [29]. The expression for $g_2 \circ \tilde{Q}$ in (5.7) consists of an infinite sum. One can see from (A.2) in appendix A the coefficients $\mathcal{C}_{0t}$ decrease. Evaluating the quadratic term $\tilde{S}^2$ for specific fields of level 0 and level 2 shows that this series is a Liebnitz series which guarantees its convergence. We demonstrate this convergence by evaluating the sum till various values of $t_{\max}$ for fields $\Phi(0) = \Lambda(0) = c(0)$.

| $t_{\max}$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
|---|---|---|---|---|---|---|---|---|
| $\langle g_1 \circ c(0) g_2(\circ \tilde{Q} c(0)\rangle$ | 4 | 6.666 | 6.133 | 6.302 | 6.243 | 6.264 | 6.257 | 6.260 |

(5.11)

It is easy to see that the series converges rapidly.

**Case 3.** $Q = c_0 + (c_2 + c_{-2})/2$
Using similar methods as in the case of $Q = c_0$, $g_2 \circ \tilde{Q}$ is given by

$$g_2 \circ \tilde{Q} = \oint \frac{dz}{2\pi i} \left( \mathcal{V}_{c_0}(z) + \frac{1}{2}(\mathcal{V}_{c_2}(z) + \mathcal{V}_{c_{-2}}(z)) \right) . \tag{5.12}$$

The current $\mathcal{V}_{c_0}$ is given in (5.8), while the currents $\mathcal{V}_{c_2}$ and $\mathcal{V}_{c_{-2}}$ are given by

$$\begin{aligned} \mathcal{V}_{c_2}(z) &= \sum_{t=2}^{\infty} \mathcal{C}_{2t} \left[ \frac{1}{\tan(2\tan^{-1} z)} \right]^{t+2} \frac{4}{(1+z^2)^2 \cos^4(2\tan^{-1} z)} , \\ \mathcal{V}_{c_{-2}}(z) &= \sum_{t=-2}^{\infty} \mathcal{C}_{(-2)t} \left[ \frac{1}{\tan(2\tan^{-1} z)} \right]^{t+2} \frac{4}{(1+z^2)^2 \cos^4(2\tan^{-1} z)} . \end{aligned} \tag{5.13}$$

The coefficients $\mathcal{C}_{-2t}$ are defined in (2.24). The prescription for the contour is the same as in the case for the current $\mathcal{V}_{c_0}$. The series resulting from the above sum are also Liebnitz series for fields till level 2. They converge rapidly. For an accuracy up to 3 places in decimal it is sufficient to retain 11 terms in the series.

### 5.1.2 The Witten vertex

The sliver limit for the Witten vertex is obtained by

$$\begin{aligned} \tilde{S}^3 &= \lim_{n\to\infty} S^3_{nnn} \\ &= \langle \tilde{f}_1 \circ \Omega(0) \tilde{f}_2 \circ \Psi(0) \tilde{f}_3 \circ \Lambda(0) \rangle . \end{aligned} \tag{5.14}$$

Taking the sliver limit for functions in (4.19) we obtain

$$\begin{aligned} \tilde{f}_1(z) &= M \left[ e^{i\pi/3} \left( \frac{1+iz}{1-iz} \right)^{1/3} \right] = \frac{1}{\sqrt{3}} + \frac{4}{9} z + \frac{4}{27\sqrt{3}} z^2 - \frac{28}{243} z^3 - \frac{20}{243\sqrt{3}} z^4 + \cdots , \\ \tilde{f}_2(z) &= M \left[ e^{i5\pi/3} \left( \frac{1+iz}{1-iz} \right)^{1/3} \right] \\ &= -\frac{1}{\sqrt{3}} + \frac{4}{9} z - \frac{4}{27\sqrt{3}} z^2 - \frac{28}{243} z^3 + \frac{20}{243\sqrt{3}} z^4 + \cdots , \\ \tilde{f}_3(z) &= M \left[ e^{i\pi} \left( \frac{1+iz}{1-iz} \right)^{1/3} \right] \\ &= -\frac{3}{z} - \frac{8}{9} z + \frac{56}{243} z^3 - \frac{776}{6561} z^5 + \frac{4456}{59049} z^7 \cdots . \end{aligned} \tag{5.15}$$

Note that it is easy to see that the vertex $\tilde{S}^3$ has cyclic symmetry. The SL$(2,\mathbb{R})$ transformation $M \circ \exp(2\pi i/3) \circ M^{-1}$ cyclically permutes the terms in the vertex.

### 5.2 Discrete symmetries of the action

The vertices $\tilde{S}^2$ and $\tilde{S}^3$ inherits the same discrete symmetries of the cubic string field theory. We show that the quadratic term vanishes for string states of different world sheet parity when the BRST operator $Q$ is of even parity. The quadratic term for excitations $\Phi(0)$ and $\Lambda(0)$ above the sliver state is given by

$$\tilde{S}^2 = \langle g_1 \circ \Phi(0)(g_2 \circ I \circ W_\infty \circ I \circ Q) g_2 \circ \Lambda(0) \,. \tag{5.16}$$

The maps $g_1, g_2, I$ have following property

$$\begin{aligned} M \circ \tilde{I} \circ M^{-1} \circ g_1(z) &= g_1 \circ P(z)\,, \\ M \circ \tilde{I} \circ M^{-1} \circ g_2(z) &= g_2 \circ P(z)\,, \\ P \circ I(z) &= I \circ P(z)\,. \end{aligned} \tag{5.17}$$

Here $P$ and $\tilde{I}$ refer to the map $P(z) = -z$ and $\tilde{I} = 1/z$. $M \circ \tilde{I} \circ M^{-1}$ is a combination of world sheet parity and $\mathrm{SL}(2,\mathbb{R})$ transformation. Applying the transformation $M \circ \tilde{I} \circ M^{-1}$ to each of the terms in the correlation function $\tilde{S}^2$ leaves it invariant as the $\mathrm{SL}(2,\mathbb{R})$ vacuum is invariant. Using this transformation we can shift the action of the parity to the fields $\Phi(0)$ and $\Lambda(0)$ and to the current $W_\infty \circ Q$. Using the explicit representation of $W_\infty \circ Q$ it is easy to see that it is left invariant if $Q$ is of even parity. The net sign picked up is the sum of parity of the two fields $\Phi(0)$ and $\Lambda(0)$. Thus the quadratic term vanishes for fields of different parity.

The quadratic term is also symmetric under exchange of $\Phi(0)$ and $\Lambda(0)$. This can be seen using the following properties of $g_1$ and $g_1$.

$$I \circ g_1(z) = g_2(z)\,, \qquad I \circ g_2(z) = g_1(z)\,. \tag{5.18}$$

Furthermore from the explicit expressions for $g_2 \circ \tilde{Q}$ in (5.7) and (5.13) and the contour used in defining the transformed BRST operator we obtain the result $I \circ g_2 \circ \tilde{Q} = -g_1 \circ \tilde{Q}$. The correlation function $\tilde{S}_2(z)$ is invariant under the action $I$ on each of the fields. This exchanges the action of $g_1$ and $g_2$ with a negative sign. Now interchanging the fields $\Phi(0)$ and $\Lambda(0)$ picks up another negative sign. Thus the quadratic term is symmetric in its two fields. There symmetry properties are borne out by explicitly calculations till level 2.

The Witten vertex for excitations above the sliver state has the following symmetry.

$$\begin{aligned} \tilde{f}_1(-z) &= M \circ \tilde{I} \circ M^{-1} \circ \tilde{f}_2(z)\,, \\ \tilde{f}_2(-z) &= M \circ \tilde{I} \circ M^{-1} \circ \tilde{f}_1(z)\,, \\ \tilde{f}_3(-z) &= M \circ \tilde{I} \circ M^{-1} \circ \tilde{f}_3(z)\,. \end{aligned} \tag{5.19}$$

Using this symmetry one can show that the cubic vertex involving fields of ghost number one with net even parity vanishes. This prohibits terms with one parity even and two parity odd fields. It also prohibits terms with three parity even fields.

This twist symmetry of the cubic vertex for excitations above the sliver is the same as that of excitations above the $\mathrm{SL}(2,\mathbb{R})$ vacuum. As mentioned before the Witten vertex for excitations above the sliver posses cyclic symmetry in its three fields. This can be seen by the $\mathrm{SL}(2,\mathbb{R})$ transformation $M \circ \exp(2\pi i/3) \circ M^{-1}$ which cyclically permutes the terms in the vertex.

### 5.3 The effective action

In this section we will apply the Witten vertex and the quadratic term derived for excitations on the sliver to evaluate the effective action. We will find the gauge unfixed action for fields representing excitations of level two on the sliver state. The level of the excitation over the sliver state is defined by the level of the operator $\mathcal{O}$ in (4.4) creating the excitation. The effective action is computed for two choices of the BRST operator. $Q = c_0$ represents a class of BRST operators which do not annihilate the identity of the string algebra and $Q = c_0 + (c_2 + c_{-2})/2$ represents the class which annihilates the identity. The string field theory action at the tachyon vacuum is given by

$$S(\Phi) = -\frac{K}{g_0^2}\left(\frac{1}{2}\langle\Phi|Q|\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right). \tag{5.20}$$

Here $Q$ represents the BRST operator at the tachyon vacuum and $K$ stands for the unknown overall normalization constant of the BRST operator. It must be possible to show from this action that one can obtain the D25-brane solution. Using an antsaz that the solution factorizes into the matter sector and the ghost sector the ratio of various D-brane tensions to a high degree of accuracy in [31]. The translational invariant solution in the matter sector used by [31] was the matter sector of the sliver state. This motivates the use of the complete sliver state including the ghost sector for the full solution. As the string field has ghost number one we require a ghost number one excitation on the sliver state. Assuming the factorization of the solution to the ghost sector and the matter sector, the excitation can be chosen to be purely from the ghost sector. Thus the D25-brane will be a represented by a local maximum in the effective action for these excitations. Let the normalization of the D25-brane tension be given by $1/(g^2 2\pi^2)$ [42]. Then the potential is defined as

$$\begin{aligned} V(\Phi) &= -g^2 2\pi^2 S(\Phi) \\ &= 2\pi^2 K\left(\frac{1}{2}\langle\Phi|Q|\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right). \end{aligned} \tag{5.21}$$

The definition of $V(\Phi)$ is chosen so that it equals 1 for the D25-brane solution. This will fix the unknown normalization constant $K$. Because of the twist invariance of the action for excitations above the sliver vacuum we can set all the odd level fields to zero. We now analyse the effective action for excitations above the sliver state for level 2 fields. The kinetic terms in all of these effective potentials are evaluated to 3 decimal places of accuracy using (5.7) and (5.13).



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**BRST operator** $\quad Q = c_0$

At level $(0,0)$ the field is just the transformed tachyon $t\tilde{c}_1$. The potential is

$$V^{(0,0)} = 2\pi^2 K \left(3.13t^2 + \frac{81}{8\sqrt{3}}t^3\right). \tag{5.22}$$

The maximum of the potential at this level is attained at the $t = -0.357$. This gives $V^{(0,0)} = 2.624K$. Now we go over to level $(2,4)$. The fields purely in the ghost sector are

$$t\tilde{c}_1|\Xi\rangle + A\tilde{c}_{-1}|\Xi\rangle + B\tilde{b}_{-2}\tilde{c}_0\tilde{c}_1|\Xi\rangle\,. \tag{5.23}$$

We do not fix the Feynman-Siegel gauge as the sliver state itself violates this gauge. The effective potential to this level is given by

$$\begin{aligned} V^{(2,4)} = 2\pi^2 K \Bigg( & 3.13t^2 + 1.956A^2 + 2.803B^2 + 4.694tA - 5.933tB - 4.450AB + \\ & + \frac{81}{8\sqrt{3}}t^3 - \frac{69\sqrt{3}}{8}t^2 A + \frac{175}{8\sqrt{3}}tA^2 - \frac{99\sqrt{3}}{8}t^2 B + \frac{1085}{24\sqrt{3}}tB^2 - \\ & - \frac{293}{4\sqrt{3}}tAB\Bigg)\,. \end{aligned} \tag{5.24}$$

A local maximum is attained at $t = -0.496, A = .114, B = .022$. The value of the tension at the potential at this point is $V^{(2,4)} = 2.614K$. It is close to the level zero value. Thus it seems that level truncation of the string field theory around the tachyon vacuum using excitations on the sliver vacuum rather than the $\mathrm{SL}(2,\mathbb{R})$ vacuum seems to converge. It is interesting to point out that level truncation of the string field theory with $Q = c_0$ using excitations on the $\mathrm{SL}(2,\mathbb{R})$ vacuum in the Feynman-Siegel gauge pushed the maximum to zero [29].

**BRST operator** $\quad Q = c_0 + (c_2 + c_{-2})/2$

We now repeat the calculation of the effective potential with the BRST operator $Q = c_0 + (c_2 + c_{-2})/2$. Only the kinetic term is modified. At level $(0,0)$ we obtain the following potential

$$V^{(0,0)} = 2\pi^2 K' \left(4.254t^2 + \frac{81}{3\sqrt{3}}t^3\right). \tag{5.25}$$

Here $K'$ stands for the different normalization constant corresponding to the BRST operator $Q = c_0 + (c_2 + c_{-2})/2$. We obtain a local maximum at $t = -0.4852$. The potential at this point is $V^{(0,0)} = 6.589K'$. At level 2 the effective potential is

$$\begin{aligned} V^{(2,4)} = 2\pi^2 K' \Bigg( & 4.254t^2 + 2.659A^2 + 3.767B^2 + 6.381tA - 8.021tB - 6.012AB + \\ & + \frac{81}{8\sqrt{3}}t^3 - \frac{69\sqrt{3}}{8}t^2 A + \frac{175}{8\sqrt{3}}tA^2 - \frac{99\sqrt{3}}{8}t^2 B + \frac{1085}{24\sqrt{3}}tB^2 - \\ & - \frac{293}{4\sqrt{3}}tAB\Bigg)\,. \end{aligned} \tag{5.26}$$

A local maximum occurs at $t = -0.675, A = 0.158, B = -0.0289$. The value of the potential at this point is $V^{(2,4)} = 6.530K'$. This also is close to the value of the potential obtained in the zeroth level approximation. Thus it looks like at least to level two there is no difference in the behaviour of the BRST operators $Q = c_0$ and $Q = c_0 + (c_2 + c_{-2})/2$.

## 6. Other excitations on the sliver state

In this section we will discuss the difficulty with another choice of excitations on the sliver state. Consider excitations of the form given by $\mathcal{O}|\Xi\rangle$ where $\mathcal{O}$ are operators built out of the ghost fields. The conventional modes of the ghost fields cannot be organized as creation and annihilation operators over the sliver state as $c_m|\Xi\rangle \neq 0$ for any value of $m$. It is the same case with the $b$ ghost modes. We will show the Witten vertex for these excitations are not well defined. The Witten vertex for these excitation can be derived using the generalized gluing theorem and the methods discussed in sections 4 and 5. It is given by

$$\begin{aligned}\bar{S}^3 &= \langle V_3|\ \Omega|\Xi\rangle \otimes \Phi|\Xi\rangle \otimes \Lambda|\Xi\rangle \\ &= \langle \bar{f}_1 \circ \Omega \bar{f}_2 \circ \Phi \bar{f}_3 \circ \Lambda\rangle\,.\end{aligned} \tag{6.1}$$

where

$$\begin{aligned}\bar{f}_1(z) &= \lim_{n\to\infty} M\left[e^{i\pi/3}\left(\frac{1+iz}{1-iz}\right)^{\frac{2}{3n-3}}\right], \\ \bar{f}_2(z) &= \lim_{n\to\infty} M\left[e^{5i\pi/3}\left(\frac{1+iz}{1-iz}\right)^{\frac{2}{3n-3}}\right], \\ \bar{f}_2(z) &= \lim_{n\to\infty} M\left[e^{i\pi}\left(\frac{1+iz}{1-iz}\right)^{\frac{2}{3n-3}}\right].\end{aligned} \tag{6.2}$$

The presence of the finite phases in the definition of the functions $\bar{f}_1(z), \bar{f}_2(z)$ and $\bar{f}_3(z)$ makes the limit $n \to \infty$ ill defined. For example the Witten vertex for the field $c_1|\Xi\rangle$ is given by

$$\begin{aligned}\bar{S}^3(c_1c_1c_1) &= \langle \bar{f}_1 \circ c_1 \bar{f}_2 \circ c_1 \bar{f}_3 \circ c_1\rangle \\ &= \lim_{n\to\infty} \frac{81\sqrt{3}}{64}(n-1)^3\,.\end{aligned} \tag{6.3}$$

The Witten vertex for these fields diverge as $n^3$. Thus these excitations are not as well defined as the ones defined by the similarity transformation discussed earlier.

## 7. Conclusions

We have developed a method to analyse excitations on wedge states and the sliver state. This is useful in exploring the solution of the string field equations around the tachyon vacuum including the ghost sector. We have used the gluing and resmoothing theorem to re-derive the product law of wedge states explicitly. The theorem was then used to construct the Witten vertex and the quadratic term for excitations on wedge states and the sliver state. We verified that the discrete symmetries for the effective action for excitations on the sliver state was inherited form the cubic action. We analyzed the gauge unfixed effective action of ghost number one excitations on the sliver till the second level for two choices of the BRST operator at the tachyon vacuum. These excitations were purely in the ghost sector. One expects the value of a local maximum of the effective potential to correspond to the tension of the D25-brane upto an overall normalization constant which depends on the choice of the BRST operator. The choice $Q = c_0$ which represents a class of BRST operators that annihilates the identity of the star algebra. For this choice we obtained that at level $(0,0)$ the value of the potential is $2.624K$ while at level $(2,4)$ it is $2.614K$. Here $K$ is the unknown normalization constant corresponding to the BRST operator $Q = c_0$. The operator $Q = c_0 + (c_2 + c_{-2})/2$ represents a class of BRST operator that annihilates the identity. The local maximum at level $(0,0)$ is given by $6.589K'$ and at level $(2,4)$ it is $6.530K'$. $K'$ is the unknown normalization constant corresponding to the BRST operator $Q = c_0 + (c_2 + c_{-2})/2$. These results indicate that level truncation using purely ghost excitations on the sliver state seem to converge for both choices of the BRST operator. It also provides evidence for the conjectured string field theory actions around the tachyon vacuum. It would be interesting to carry this analysis of the effective action for excitations purely ghost sector over the sliver vacuum to a higher level. We note that there are excitations on the sliver state for which the Witten vertex is not well defined.

## Acknowledgments

The author is especially grateful to Ashoke Sen for the many useful discussions comments on the manuscript and encouragement. He thanks Partha Mukhopadhyay and Barton Zweibach for useful discussions. He is grateful to Spenta Wadia for encouragement. The work of the author is supported by NSF grant PHY97-2202.

## A. Similarity transformation $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$

The expression for the width matrix $S$ given in (2.14) requires the matrices $\mathcal{A}$ representing the similarity transformation of $\alpha_m^\mu$ with the operator $U_{W_\infty}$. We list a few coefficients of the matrix $\mathcal{A}$ below.

$$
\begin{aligned}
U_{W\infty}\alpha^{\mu}_{-1}U^{-1}_{W_\infty} &= \alpha^{\mu}_{-1} - \frac{1}{3}\alpha^{\mu}_{1} - \frac{1}{45}\alpha^{\mu}_{3} - \frac{2}{945}\alpha^{\mu}_{5} - \frac{1}{4725}\alpha^{\mu}_{7} + \cdots\,,\\
U_{W\infty}\alpha^{\mu}_{-2}U^{-1}_{W_\infty} &= \alpha^{\mu}_{-2} - \frac{2}{3}\alpha^{\mu}_{0} + \frac{1}{15}\alpha^{\mu}_{2} + \frac{2}{189}\alpha^{\mu}_{4} + \frac{1}{675}\alpha^{\mu}_{6} + \frac{2}{10395}\alpha^{\mu}_{8} + \cdots\,,\\
U_{W\infty}\alpha^{\mu}_{-3}U^{-1}_{W_\infty} &= \alpha^{\mu}_{-3} - \alpha^{\mu}_{-1} + \frac{4}{15}\alpha^{\mu}_{1} + \frac{1}{945}\alpha^{\mu}_{3} - \frac{11}{4725}\alpha^{\mu}_{5} - \frac{29}{51975}\alpha^{\mu}_{7} + \cdots\,,\\
U_{W\infty}\alpha^{\mu}_{-4}U^{-1}_{W_\infty} &= \alpha^{\mu}_{-4} - \frac{4}{3}\alpha^{\mu}_{-2} + \frac{26}{25}\alpha^{\mu}_{0} - \frac{64}{945}\alpha^{\mu}_{2} - \frac{19}{2835}\alpha^{\mu}_{4} - \frac{4}{22275}\alpha^{\mu}_{6} + \cdots\,. \qquad \text{(A.1)}
\end{aligned}
$$

The matrix $\mathcal{C}$ represents the similarity transformation of the ghost modes $c_m$ by the operator $U_{W_\infty}$. This is required to evaluate the width matrix $\tilde{S}$ of the ghost sector given in (2.16) and (2.17). We write down a few coefficients of $\mathcal{C}$ below.

$$
\begin{aligned}
U_{W_\infty}c_2U^{-1}_{W_\infty} &= c_2 + 2c_4 + \frac{7}{3}c_6 + \frac{94}{45}c_8 + \cdots\,,\\
U_{W_\infty}c_1U^{-1}_{W_\infty} &= c_1 + \frac{5}{3}c_3 + \frac{74}{45}c_5 + \frac{239}{189}c_7 + \cdots\,,\\
U_{W_\infty}c_0U^{-1}_{W_\infty} &= c_0 + \frac{4}{3}c_2 + \frac{16}{15}c_4 + \frac{128}{189}c_6 + \frac{256}{675}c_8 \cdots\,,\\
U_{W_\infty}c_{-1}U^{-1}_{W_\infty} &= c_{-1} + c_1 + \frac{3}{5}c_3 + \frac{274}{945}c_5 + \frac{599}{4725}c_7 + \cdots\,,\\
U_{W_\infty}c_{-2}U^{-1}_{W_\infty} &= c_{-2} + \frac{2}{3}c_0 + \frac{11}{45}c_2 + \frac{62}{945}c_4 + \frac{41}{2835}c_6 + \frac{62}{22275}c_8 + \cdots\,,\\
U_{W_\infty}c_{-3}U^{-1}_{W_\infty} &= c_{-3} + \frac{1}{3}c_{-1} - \frac{31}{945}c_3 - \frac{41}{2835}c_5 - \frac{31}{7425}c_7 + \cdots\,,\\
U_{W_\infty}c_{-4}U^{-1}_{W_\infty} &= c_{-4} - \frac{2}{15}c_0 - \frac{8}{189}c_2 - \frac{1}{225}c_4 + \frac{8}{6237}c_6 + \frac{2218}{2764125}c_8 + \cdots\,, \qquad \text{(A.2)}\\
U_{W_\infty}c_{-5}U^{-1}_{W_\infty} &= c_{-5} - \frac{1}{3}c_{-3} - \frac{7}{45}c_{-1} + \frac{176}{14175}c_3 + \frac{53}{13365}c_5 + \frac{17917}{30405375}c_7 + \cdots\,.
\end{aligned}
$$

We also need the similarity transformation of the modes of the $b$ ghost by $U_{W_\infty}$. These are listed below.

$$
\begin{aligned}
U_{W_\infty}b_{-2}U^{-1}_{W_\infty} &= b_{-2} - \frac{4}{3}b_0 + \frac{29}{45}b_2 - \frac{128}{945}b_4 + \cdots\,,\\
U_{W_\infty}b_{-3}U^{-1}_{W_\infty} &= b_{-3} - \frac{5}{3}b_{-1} + \frac{16}{15}b_1 - \frac{61}{189}b_3 + \frac{31}{675}b_5 + \cdots\,,\\
U_{W_\infty}b_{-4}U^{-1}_{W_\infty} &= b_{-4} - 2b_{-2} + \frac{8}{5}b_0 - \frac{608}{945}b_2 + \frac{629}{4725}b_4 + \cdots\,. \qquad \text{(A.3)}
\end{aligned}
$$

## B. Conformal transformations

To evaluate correlation function with operators non-primary $c_0$, $c_{-1}$ and $b_{-2}c_0c_1$ we need the following conformal transformations, respectively.

$$
f \circ \partial c(z) = -\frac{f''(z)}{(f'^2)(z)}c(f(z)) + \partial c(f(z))\,,
$$

$$
\begin{aligned}
f \circ \frac{\partial^2 c(z)}{2} &= \left[2\frac{(f''(z))^2}{(f'(z))^3} - \frac{f'''(z)}{(f'(z))^2}\right]\frac{c(f(z))}{2} - \frac{f''(z)}{f'}\frac{\partial c(f(z))}{2} + \\
&\quad + f'(z)\frac{\partial^2 c(f(z))}{2}\,, \\
f \circ b\partial cc(z) &= f'(z) b\partial cc(f(z)) + \left[\frac{2}{3}\frac{f'''(z)}{(f'(z))^2} - \frac{1}{4}\frac{(f''(z))^2}{(f'(z))^3}\right] c(f(z)) - \\
&\quad - \frac{3}{2}\frac{f''(z)}{f'(z)}\partial c(f(z))\,.
\end{aligned}
\tag{B.1}
$$

## References


[1] E. Witten, *Noncommutative geometry and string field theory*, *Nucl. Phys.* **B 268** (1986) 253.

[2] A. Sen, *Descent relations among bosonic D-branes*, *Int. J. Mod. Phys.* **A 14** (1999) 4061 [hep-th/9902105].

[3] A. Sen, *Non-BPS states and branes in string theory*, hep-th/9904207.

[4] K. Bardakci and M.B. Halpern, *Explicit spontaneous breakdown in a dual model*, *Phys. Rev.* **D 10** (1974) 4230;
K. Bardakci, *Spontaneous symmetry breakdown in the standard dual string model*, *Nucl. Phys.* **B 133** (1978) 297.

[5] V.A. Kostelecky and S. Samuel, *The static tachyon potential in the open bosonic string theory*, *Phys. Lett.* **B 207** (1988) 169; *On a nonperturbative vacuum for the open bosonic string*, *Nucl. Phys.* **B 336** (1990) 263.

[6] A. Sen and B. Zwiebach, *Tachyon condensation in string field theory*, *J. High Energy Phys.* **03** (2000) 002 [hep-th/9912249].

[7] W. Taylor, *D-brane effective field theory from string field theory*, *Nucl. Phys.* **B 585** (2000) 171 [hep-th/0001201].

[8] N. Moeller and W. Taylor, *Level truncation and the tachyon in open bosonic string field theory*, *Nucl. Phys.* **B 583** (2000) 105 [hep-th/0002237].

[9] J.A. Harvey and P. Kraus, *D-branes as unstable lumps in bosonic open string field theory*, *J. High Energy Phys.* **04** (2000) 012 [hep-th/0002117].

[10] R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, *Lumps and p-branes in open string field theory*, *Phys. Lett.* **B 482** (2000) 249 [hep-th/0003031].

[11] N. Moeller, A. Sen and B. Zwiebach, *D-branes as tachyon lumps in string field theory*, *J. High Energy Phys.* **08** (2000) 039 [hep-th/0005036].

[12] J.R. David, U(1) *gauge invariance from open string field theory*, *J. High Energy Phys.* **10** (2000) 017 [hep-th/0005085].

[13] L. Rastelli and B. Zwiebach, *Tachyon potentials, star products and universality*, hep-th/0006240.

[14] A. Sen and B. Zwiebach, *Large marginal deformations in string field theory*, *J. High Energy Phys.* **10** (2000) 009 [hep-th/0007153].

[15] W. Taylor, *Mass generation from tachyon condensation for vector fields on D-branes*, *J. High Energy Phys.* **08** (2000) 038 [hep-th/0008033].

[16] R. de Mello Koch and J.P. Rodrigues, *Lumps in level truncated open string field theory*, *Phys. Lett.* **B 495** (2000) 237 [hep-th/0008053].

[17] N. Moeller, *Codimension two lump solutions in string field theory and tachyonic theories*, hep-th/0008101.

[18] V.A. Kostelecky and R. Potting, *Analytical construction of a nonperturbative vacuum for the open bosonic string*, *Phys. Rev.* **D 63** (2001) 046007 [hep-th/0008252].

[19] H. Hata and S. Shinohara, *Brst invariance of the non-perturbative vacuum in bosonic open string field theory*, *J. High Energy Phys.* **09** (2000) 035 [hep-th/0009105].

[20] B. Zwiebach, *Trimming the tachyon string field with* $\mathrm{SU}(1,1)$, hep-th/0010190.

[21] M. Schnabl, *Constraints on the tachyon condensate from anomalous symmetries*, *Phys. Lett.* **B 504** (2001) 61 [hep-th/0011238].

[22] P. Mukhopadhyay and A. Sen, *Test of Siegel gauge for the lump solution*, *J. High Energy Phys.* **02** (2001) 017 [hep-th/0101014].

[23] H. Hata and S. Teraguchi, *Test of the absence of kinetic terms around the tachyon vacuum in cubic string field theory*, *J. High Energy Phys.* **05** (2001) 045 [hep-th/0101162].

[24] B. Feng, Y.-H. He and N. Moeller, *Testing the uniqueness of the open bosonic string field theory vacuum*, hep-th/0103103.

[25] I. Ellwood, B. Feng, Y.-H. He and N. Moeller, *The identity string field and the tachyon vacuum*, hep-th/0105024.

[26] I. Ellwood and W. Taylor, *Gauge invariance and tachyon condensation in open string field theory*, hep-th/0105156.

[27] I. Ellwood and W. Taylor, *Open string field theory without open strings*, *Phys. Lett.* **B 512** (2001) 181 [hep-th/0103085].

[28] K. Ohmori, *A review on tachyon condensation in open string field theories*, hep-th/0102085.

[29] L. Rastelli, A. Sen and B. Zwiebach, *String field theory around the tachyon vacuum*, hep-th/0012251.

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[30] G.T. Horowitz, J. Morrow-Jones, S.P. Martin and R.P. Woodard, *New exact solutions for the purely cubic bosonic string field theory*, *Phys. Rev. Lett.* **60** (1988) 261.

[31] L. Rastelli, A. Sen and B. Zwiebach, *Classical solutions in string field theory around the tachyon vacuum*, `hep-th/0102112`.

[32] L. Rastelli, A. Sen and B. Zwiebach, *Half strings, projectors and multiple D-branes in vacuum string field theory*, `hep-th/0105058`.

[33] D.J. Gross and W. Taylor, *Split string field theory, 1*, `hep-th/0105059`.

[34] T. Kawano and K. Okuyama, *Open string fields as matrices*, *J. High Energy Phys.* **06** (2001) 061 [`hep-th/0105129`].

[35] A. Dhar, G. Mandal and S.R. Wadia, *Nonrelativistic fermions, coadjoint orbits of w(infinity) and string field theory at $c = 1$*, *Mod. Phys. Lett.* **A 7** (1992) 3129 [`hep-th/9207011`]; *$W_\infty$ coherent states and path integral derivation of bosonization of nonrelativistic fermions in one-dimension*, *Mod. Phys. Lett.* **A 8** (1993) 3557 [`hep-th/9309028`]; *String field theory of two-dimensional QCD: a realization of $W_\infty$ algebra*, *Phys. Lett.* **B 329** (1994) 15 [`hep-th/9403050`].

[36] L. Rastelli, A. Sen and B. Zwiebach, *Boundary CFT construction of D-branes in vacuum string field theory*, `hep-th/0105168`.

[37] D.J. Gross and A. Jevicki, *Operator formulation of interacting string field theory*, *Nucl. Phys.* **B 283** (1987) 1; *Operator formulation of interacting string field theory, 2*, *Nucl. Phys.* **B 287** (1987) 225.

[38] S. Samuel, *On the string field theory proposal of E. Witten*, *Phys. Lett.* **B 181** (1986) 255.

[39] N. Ohta, *Covariant interacting string field theory in the fock space representation*, *Phys. Rev.* **D 34** (1986) 3785, erratum *ibid.* **D35** (2627) 1987.

[40] A. LeClair, M.E. Peskin and C.R. Preitschopf, *String field theory on the conformal plane, 1. Kinematical principles*, *Nucl. Phys.* **B 317** (1989) 411; *String field theory on the conformal plane, 2. Generalized gluing*, *Nucl. Phys.* **B 317** (1989) 464.

[41] A.S. Schwarz and A. Sen, *Gluing theorem, star product and integration in open string field theory in arbitrary background fields*, *Int. J. Mod. Phys.* **A 6** (1991) 5387.

[42] A. Sen, *Universality of the tachyon potential*, *J. High Energy Phys.* **12** (1999) 027 [`hep-th/9911116`].